Study Wind Tunnel Calculator
Key Formulae, Theory, and Section-by-Section Loss Modelling

1 Conventions and scope

This document defines the geometry, boundary-layer blockage, and loss-modelling relations for an open-circuit study wind tunnel consisting of:

$$\text{Contraction} \;\rightarrow\; \text{Test section} \;\rightarrow\; \text{Diffuser.}$$

Throughout:

$$X = \text{width}, \qquad Y = \text{height}, \qquad Z = \text{length (streamwise)}.$$

2 Nomenclature

Symbol	Meaning
X, Y, Z	width, height, length [m]
A	cross-sectional area [m^2]
R_c	contraction area ratio $R_c = \dfrac{A_{c,i}}{A_{c,o}}$
R_t	test-section side ratio $R_t = \dfrac{Y_t}{X_t}$
R_d	diffuser area ratio $R_d = \dfrac{A_{d,o}}{A_{d,i}}$
θ_e	diffuser expansion angle [deg]
γ	ratio of specific heats
R	specific gas constant [J kg^{-1} K^{-1}]
T	static temperature [K]
a	speed of sound $a = \sqrt{\gamma RT}$ [m s^{-1}]
M	Mach number
M_t	Mach number in test section
M_i	average Mach number in section i
V	flow speed [m s^{-1}]
$\dot{V}$	volumetric flow rate [m^3 s^{-1}]
ρ	density [kg m^{-3}]
ν	kinematic viscosity [m^2 s^{-1}]
D_h	hydraulic diameter [m]
D_t	test-section hydraulic diameter
Re_t	reference Reynolds number for test section
Re_i	Reynolds number in section i
Re_{zt}	Reynolds number based on Z_t: $\dfrac{V_t Z_t}{\nu}$
f_i	Darcy friction factor in section i
δ	boundary-layer thickness [m]
BL	blockage ratio (boundary-layer blockage)
q	dynamic pressure $q = \frac{1}{2}\rho V^2$ [Pa]
ΔP	pressure drop [Pa]
k	loss coefficient (dimensionless)
$k_{L,t}$	loss coefficient defined by $k_{L,t} = \Delta H_L / q_t$
P_t	test-section power scale [W]
P_c	total power loss rate around tunnel [W]
E_r	energy ratio $E_r = \dfrac{P_t}{P_c}$

3 Geometry

3.1 Test section

$$R_t = \frac{Y_t}{X_t}. \tag{1}$$

$$A_t = X_t Y_t = (X_t)^2 R_t, \qquad Y_t = X_t R_t, \qquad Z_t = 2X_t. \tag{2}$$

3.2 Contraction section (assuming square inlet)

$$R_c = \frac{A_{c,i}}{A_{c,o}}. \tag{3}$$

$$A_{c,o} = A_t, \qquad A_{c,i} = A_{c,o} R_c = A_t R_c. \tag{4}$$

$$X_{c,i} = \sqrt{A_{c,i}} = \sqrt{A_t R_c}, \qquad Y_{c,i} = X_{c,i}. \tag{5}$$

$$X_{c,o} = X_t, \qquad Y_{c,o} = Y_t. \tag{6}$$

$$Z_c = X_{c,i}, \qquad X_{c,\text{setting}} = \frac{X_{c,i}}{2} = \frac{\sqrt{A_t R_c}}{2}. \tag{7}$$

3.3 Diffuser section (square outlet)

$$R_d = \frac{A_{d,o}}{A_{d,i}}. \tag{8}$$

$$A_{d,i} = A_t, \qquad A_{d,o} = A_{d,i} R_d = A_t R_d, \tag{9}$$

$$X_{d,i} = X_t, \quad Y_{d,i} = Y_t, \qquad X_{d,o} = \sqrt{A_{d,o}} = \sqrt{A_t R_d}, \quad Y_{d,o} = X_{d,o}. \tag{10}$$

$$Z_d = \frac{Y_{d,o} - Y_t}{2\tan(\theta_e)} = \frac{X_{d,o} - X_t R_t}{2\tan(\theta_e)}. \tag{11}$$

$$Z_d = \frac{X_{d,o} - X_t R_t}{2\tan(\theta_e)} = \frac{\sqrt{A_t R_d} - X_t R_t}{2\tan(\theta_e)} = \frac{\sqrt{(R_t X_t^2) R_d} - X_t R_t}{2\tan(\theta_e)} = \frac{X_t \left(\sqrt{R_t R_d} - R_t\right)}{2\tan(\theta_e)}.$$

3.4 Total tunnel length

$$Z_w = \frac{X_t}{2}\left(3\sqrt{\frac{R_t}{R_c}} + 4 + \frac{\sqrt{R_d R_t} - 1}{\tan(\theta_e)}\right). \tag{12}$$

4 Volumetric flow rate

$$\dot{V}_t = A_t \left(M_t \sqrt{\gamma R T}\right), \tag{13}$$

where M_t is the Mach number in the test section and $\sqrt{\gamma R T}$ is the speed of sound.

5 Boundary-layer interference and blockage

$$A_{BL} = 2\delta(X_t + Y_t). \tag{14}$$

$$A_{BL} = 2\delta X_t (1 + R_t). \tag{15}$$

$$A_t = R_t X_t^2. \tag{16}$$

$$BL = \frac{A_{BL}}{A_t} = \frac{2\delta(1 + R_t)}{R_t X_t}. \tag{17}$$

5.1 Define Re_{zt}

$$Re_{zt} = \frac{V_t Z_t}{\nu}. \tag{18}$$

5.2 Laminar wall boundary layers (Blasius), $Re_{zt} < 500000$

$$\delta = \frac{5 Z_t}{\sqrt{Re_{zt}}}. \tag{19}$$

$$BL_{\text{lam}} = \frac{10 Z_t (1 + R_t)}{R_t X_t \sqrt{Re_{zt}}}. \tag{20}$$

5.3 Turbulent wall boundary layers (1/7th-power law), $Re_{zt} > 500000$

$$\delta = \frac{0.16 Z_t}{Re_{zt}^{1/7}}. \tag{21}$$

$$BL_{\text{turb}} = \frac{0.32 Z_t (1 + R_t)}{R_t X_t Re_{zt}^{1/7}}. \tag{22}$$

5.4 Validity criterion

$$BL \leq 0.07. \tag{23}$$

6 Section losses: definitions and test-section referencing

There will be equal losses in static head and total head due to viscous action between flowing gas and solid boundaries.

For each tunnel section, the pressure loss is written in the form

$$\Delta P_i = k_i \left(\tfrac{1}{2} \rho_i V_i^2\right). \tag{24}$$

The fan must provide an equal pressure rise to balance cumulative losses:

$$|\Delta P_1| + |\Delta P_2| + |\Delta P_3| + \cdots + |\Delta P_n| = |\Delta P_{\text{fan}}|. \tag{25}$$

Non-dimensionalise the total head loss ΔH_L using the local dynamic pressure q_L:

$$k_L = \frac{\Delta H_L}{q_L} = \frac{\Delta H_L}{\left(\frac{1}{2}\right) \rho_L V_L^2}, \qquad \Delta H_L = k_L q_L. \tag{26}$$

Energy loss rate due to pressure drop:

$$\Delta \dot{E}_L = A_L V_L \Delta H_L = \dot{V}_L \Delta H_L. \tag{27}$$

Since $A_L V_L = \dot{m}_L / \rho_L$,

$$\Delta \dot{E}_L = \left(\frac{\dot{m}_L}{\rho_L}\right) k_L q_L = \left(\frac{\dot{m}_L}{\rho_L}\right) k_L \left(\tfrac{1}{2}\right) \rho_L V_L^2 = k_L \left(\tfrac{1}{2} \dot{m}_L V_L^2\right). \tag{28}$$

Define:

$$k_{L,t} = \frac{\Delta H_L}{q_t}. \tag{29}$$

Define the test-section power scale:

$$P_t = \tfrac{1}{2} \dot{m}_t V_t^2. \tag{30}$$

Then:

$$\Delta \dot{E}_L = k_{L,t} P_t, \qquad P_c = \sum k_{L,t} P_t. \tag{31}$$

Energy ratio:

$$\boxed{E_r = \frac{P_t}{P_c} = \frac{1}{\sum k_{L,t}}.} \tag{32}$$

7 Friction factor workflow (local $M \to Re \to f$)

For section i, determine:

$$M_i \;\to\; Re_i \;\to\; f_i.$$

7.1 Local Mach number (iterative; unchanged)

$$M_i = \frac{M_t}{(1+0.2M_t^2)^3}\left(\frac{A_t}{A_i}\right)\left(1+0.2M_i^2\right)^3. \tag{33}$$

$$A_i = \frac{A_{i,\text{in}} + A_{i,\text{out}}}{2}. \tag{34}$$

7.2 Local Reynolds number (unchanged)

$$Re_i = Re_t\left(\frac{L_i}{D_t}\right)\left(\frac{A_t}{A_i}\right)\left(\frac{1+\dfrac{\gamma-1}{2}M_i^2}{1+\dfrac{\gamma-1}{2}M_t^2}\right)^{0.76}. \tag{35}$$

7.3 Prandtl universal law of friction (iterative; unchanged)

$$f_i = \left[2\log_{10}\left(Re_i\sqrt{f_i}\right) - 0.8\right]^{-2}. \tag{36}$$

8 Hydraulic diameter

$$D_h = \frac{2XY}{X+Y}. \tag{37}$$

$$D_t = \frac{2X_tY_t}{X_t+Y_t}. \tag{38}$$

9 Section-by-section: explicit M_i, Re_i, f_i, and loss coefficient

9.1 Test section

$$A_t = X_tY_t = R_tX_t^2, \qquad L_t = Z_t = 2X_t, \qquad D_t = \frac{2X_tY_t}{X_t+Y_t}. \tag{39}$$

$$A_i = A_t. \tag{40}$$

$$M_t = \text{specified}. \tag{41}$$

$$Re_{zt} = \frac{V_tZ_t}{\nu}. \tag{42}$$

$$f_t = \left[2\log_{10}\left(Re_t\sqrt{f_t}\right) - 0.8\right]^{-2}. \tag{43}$$

$$k_t = f_t\frac{Z_t}{D_t}. \tag{44}$$

9.2 Diffuser

$$A_{d,i} = A_t, \qquad A_{d,o} = A_tR_d, \qquad A_d = \frac{A_{d,i}+A_{d,o}}{2} = \frac{(1+R_d)A_t}{2}, \tag{45}$$

$$L_d = Z_d = \frac{Y_{d,o}-Y_t}{2\tan(\theta_e)}. \tag{46}$$

$$M_d = \frac{M_t}{(1+0.2M_t^2)^3}\left(\frac{A_t}{A_d}\right)\left(1+0.2M_d^2\right)^3. \tag{47}$$

$$Re_d = Re_t\left(\frac{Z_d}{D_t}\right)\left(\frac{A_t}{A_d}\right)\left(\frac{1+\dfrac{\gamma-1}{2}M_d^2}{1+\dfrac{\gamma-1}{2}M_t^2}\right)^{0.76}. \tag{48}$$

$$f_d = \left[2\log_{10}\left(Re_d\sqrt{f_d}\right) - 0.8\right]^{-2}. \tag{49}$$

$$k_d = k_f + k_{ex}. \tag{50}$$

$$k_f = \left(1 - \frac{1}{R_d^2}\right)\frac{f_d}{8\sin(\theta_e)}. \tag{51}$$

$$k_{ex} = k_e(\theta_e)\left(\frac{R_d - 1}{R_d}\right)^2. \tag{52}$$

$$k_e(\theta_e) = \begin{cases} 0.09623 - 0.004152\,\theta_e, & 0 < \theta_e < 1.5, \\ 0.1222 - 0.04590\,\theta_e + 0.02203\,\theta_e^2 + 0.003269\,\theta_e^3 & \\ \qquad - 0.0006145\,\theta_e^4 - 0.00002800\,\theta_e^5 + 0.00002337\,\theta_e^6, & 1.5 < \theta_e < 5, \\ -0.01322 + 0.05866\,\theta_e, & \theta_e > 5. \end{cases} \tag{53}$$

9.3 Contraction

$$A_{c,o} = A_t, \qquad A_{c,i} = A_t R_c, \qquad A_c = \frac{A_{c,i} + A_{c,o}}{2} = \frac{(R_c + 1)A_t}{2}, \tag{54}$$

$$L_c = Z_c = X_{c,i} = \sqrt{A_t R_c}. \tag{55}$$

$$M_c = \frac{M_t}{(1 + 0.2M_t^2)^3}\left(\frac{A_t}{A_c}\right)\left(1 + 0.2M_c^2\right)^3. \tag{56}$$

$$Re_c = Re_t\left(\frac{Z_c}{D_t}\right)\left(\frac{A_t}{A_c}\right)\left(\frac{1 + \frac{\gamma - 1}{2}M_c^2}{1 + \frac{\gamma - 1}{2}M_t^2}\right)^{0.76}. \tag{57}$$

$$f_c = \left[2\log_{10}\left(Re_c\sqrt{f_c}\right) - 0.8\right]^{-2}. \tag{58}$$

$$k_c = 0.32\,\frac{f_c L_c}{D_{h,i}}. \tag{59}$$

$$D_{h,i} = \frac{2X_{c,i}Y_{c,i}}{X_{c,i} + Y_{c,i}} = X_{c,i}. \tag{60}$$

$$k_c = 0.32\,f_c. \tag{61}$$

10 Comparison with existing wind tunnel facilities

Table 1 compares energy ratios computed using the present method with measured values reported for a range of existing low-speed wind tunnel facilities.

The comparison shows that, for the majority of facilities, the computed energy ratios agree well with measured values. Larger discrepancies occur primarily in tunnels with non-standard layouts or additional elements not accounted for in the simplified loss model. Overall, the agreement demonstrates that the method provides a reliable basis for preliminary wind tunnel design and for sizing the motor required to achieve a specified test-section speed.

Table 1: Measured and computed energy ratios for selected wind tunnel facilities

Facility	Test-section speed (m/s)	Measured E_r	Computed E_r	Difference (%)
NASA Ames, 40 × 80 ft	107.3	7.88	7.96	1.0
NASA Ames, 7 × 10 ft	133.0	7.85	8.07	2.8
Lockheed Martin low-speed wind tunnel	52.3	1.10	1.12	1.8
Indian Institute of Science, 14 × 9 ft	96.3	6.85	6.83	−0.3
Hawker Siddeley Aviation, 15-ft V/STOL	45.7	2.38	3.97	66.8
University of Washington, 8 × 12 ft	117.7	8.30	7.20	−13.3
NASA Langley, 30 × 60 ft	52.7	3.71	4.73	27.4

The quoted energy ratios are the best available and best reported values for each facility.